

Math 3400 Worksheet

Monday, September 11, 2006

1. Consider the following system of linear equations:

$$\begin{aligned}2x + 4y + z &= 4 \\ -2x + 2y - 3z &= 12 \\ x + 5y - 2z &= 3\end{aligned} .$$

The augmented matrix representing this system is

$$\left(\begin{array}{ccc|c} 2 & 4 & 1 & 4 \\ -2 & 2 & -3 & 12 \\ 1 & 5 & -2 & 3 \end{array} \right).$$

After a few matrix operations we end up with

$$\left(\begin{array}{ccc|c} 1 & 2 & \frac{1}{2} & 2 \\ 0 & 1 & -\frac{1}{3} & \frac{8}{3} \\ 0 & 3 & -2.5 & 1 \end{array} \right).$$

The matrix in reduced row echelon form is

$$\left(\begin{array}{ccc|c} 1 & 2 & \frac{1}{2} & 2 \\ 0 & 1 & -\frac{1}{3} & \frac{8}{3} \\ -- & -- & -- & -- \end{array} \right).$$

The solution to our system of equations is ...

2. Suppose we were given the following system of equations in 6 variables:

$$\begin{aligned}x_1 + 2x_2 + 0x_3 - 3x_4 + x_5 + 0x_6 &= 2 \\x_1 + 2x_2 + x_3 - 3x_4 + x_5 + 2x_6 &= 3 \\x_1 + 2x_2 - 3x_4 + 2x_5 + x_6 &= 4 \\3x_1 + 6x_2 + x_3 - 9x_4 + 4x_5 + 3x_6 &= 9\end{aligned}$$

The augmented matrix for this system is

$$\left(\begin{array}{cccccc|c} 1 & 2 & 0 & -3 & 1 & 0 & 2 \\ 1 & 2 & 1 & -3 & 1 & 2 & 3 \\ 1 & 2 & 0 & -3 & 2 & 1 & 4 \\ 3 & 6 & 1 & -9 & 4 & 3 & 9 \end{array} \right).$$

After one set of matrix operations we get:

$$\left(\begin{array}{cccccc|c} 1 & 2 & 0 & -3 & 1 & 0 & 2 \\ -- & -- & -- & -- & -- & -- & -- \\ -- & -- & -- & -- & -- & -- & -- \\ -- & -- & -- & -- & -- & -- & -- \end{array} \right).$$

After several row operations we end up with

$$\left(\begin{array}{cccccc|c} 1 & 2 & \frac{1}{3} & -3 & \frac{4}{3} & 1 & 3 \\ 0 & 0 & 1 & 0 & -\frac{1}{2} & \frac{3}{2} & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right).$$

The solution to our system of equations is