Dice-Shaking as an Analogy for Radioactive Decay and First-Order Kinetics

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A very simple and easily understood experiment involving dice-shaking can be used as an analogy for radioactive decay. This exercise not only works as a qualitative analogy of the concept of nuclear transmutation, but can also be extended to describe the mathematical description of the process of first-order radioactive decay. This exercise can easily be coupled to the determination of the half-life of a radioactive isotope, ^{137m}Ba, and both exercises can be accomplished in one 3-hour laboratory period.

Description of the Analogy and the Experiment

Different sided dice (4, 6, 8, 10, 12, and 20 faces) can be obtained from most novelty stores (Fig. 1). (I will use the plural form, dice, throughout even though the singular form of dice is die.)

The analogy can most easily be understood by comparing what can be expected if two sets of dice with different numbers of faces (6 vs. 12, for instance) are shaken. I usually do this as a demonstration dialog as a way of introducing the experiment. The following question is posed: "If I pick an arbitrary number, start shaking the dice, and remove any dice that come up with the selected number, which set of dice will I finish shaking first?" There is universal agreement that, *on the average,* fewer shakes will be required for the set of dice having fewer faces. It must, however, be established that once in awhile when this game is played sets of dice with more faces may "prevail" (this type of event of course will occur during experimentation and must be rationalized). It is also important for students to recognize that essentially the same result should be obtained from playing the game once with 100 dice as from playing the game ten times with 10 dice.

The concept of half-life is introduced in the context of the dice-shaking experiment. The half-life is defined as the number of shakes required to reduce the number of dice that you have at any given time by half. The only problem here is that there is recognition of the fact that for small sets of dice this number would vary greatly. However for large sets, the "half-life" should be about the same. The analogy to nuclear transformation can now follow. The level and extent of the explanation will depend upon the student clientele in the lab, but will have the following essentials. Unstable nuclei are like dice in that when a certain "arrangement" (number) occurs, then the nucleus will change, radiation will be produced and that particular nucleus will no longer be in the "game". It is important to emphasize that one can predict neither which individual dice will have its number come up, nor which nucleus in a collection of nuclei will decay at any given moment. It follows that the set of 4-sided dice would represent a type of isotope that is fairly unstable (1 of the 4 "states" is unstable), whereas the set of 20-sided dice would represent an isotope that is not as unstable (only 1 of 20 "states" is unstable). The idea of half-life is now applied here: what is the time required for the number of nuclei present at any given time to be reduced by half?

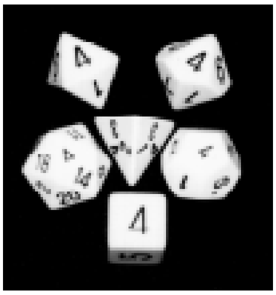

Figure 1. Four-, 6-, 8-, 10-, 12-, and 20-sided dice. All dice show the number 4 having been shaken.

Mechanics of the Experiment

The experiment consists of two parts. For the determination of the half-life of ^{137m}Ba, the Nucleus Minicounter is used. The other part involves shaking dice. The actual time spent in experimentation is rather limited. It is best if students work in pairs and interchange the roles of experimenter and recorder. Student pairs rotate between tasks on the basis of availability. A certain cooperation sets in after awhile, particularly as students seek out the higher-order dice (which in this experiment are the limiting reagent to completion).

The determination of the half-life of ^{137m}Ba is a standard experiment in many general chemistry laboratories. This experiment has been described elsewhere (*1*). A brief description of the salient points is as follows. First, the level of background radiation over a 5-min interval is determined and is recorded on a data sheet (or in a lab notebook). A sample of ^{137m}Ba is generated using a cow, and the amount of radiation in each 0.5-min interval is determined and recorded. Then the background radiation per 0.5 min is subtracted from each experimental value. The final experimental values to be plotted are total elapsed time (x) vs. corrected counts per each successive 0.5 minute interval (y).

The dice-shaking is freeform. The rules of this "experimental game" are very straightforward:

1. Obtain a set of six dice of a certain type (4-, 6-, 8-, 10-, 12-, or 20-faced) and a data sheet.
2. Pick any number that could come up on a certain set of dice and record this number and the type of dice on the sheet. (The data sheet has columns of numbers running from 1 to [1]0, repeated 4 times per column; there are 4 columns per data sheet. In this way the same data sheet can be used for any set of dice.)
3. Start shaking the dice.
4. If the selected number "comes up" on any of the dice, an X (or X's) is placed next to the number representing the shake that has been taken and these dice are removed from play; otherwise a check mark is made indicating that a shaking event has occurred.
5. The remaining dice are shaken until the selected number has come up on all the dice.
6. The experiment is repeated three more times with

this set, and then the same experiment is repeated with a different set of dice. For a class of 20, the total number of each type of dice shaken is 240 (10 groups × 6 dice × 4 trials).

Data Treatment

The numbers from the radioactivity component of the experiment are already in spreadsheet form. They can be entered into a spreadsheet program in a computer and a graphical output is obtained within minutes. For the sake of normalization and comparison to other sets of data it is useful to convert the counts per 0.5-min interval values to a fractional basis (fraction unconverted).

The numbers from the shaking component will work only if a sizable set of data is available. Therefore data must be pooled. In effect the alternative statistical scenario of shaking 240 dice in one experiment (vs. 6 dice in 40 experiments) is presented. Students can enter their data into a computer as these are obtained and the combined data can be periodically updated. The pooled raw data are converted to the form of dice still in play vs. shaking events. In this case also it is useful to normalize the value for dice still in play to a fractional value for comparison purposes.

Typical results for the shaking experiment with 4-, 8-, and 12-sided dice are shown in Figure 2. There is a very nice reinforcement of the intuitive idea that it will take longer to finish the experiment with the dice that have more sides. From the obvious difference in the pitch of the curves, it follows that the “half-life” will increase as the number of sides increases. In a bit of serendipity it is found that the “decay curve” for the 4-sided dice and that of ^{137m}Ba essentially overlap (Fig. 3). The half-life of 4-sided dice (2.7, in number of shakes) is within experimental error of the half-life of ^{137m}Ba (2.6, in minutes). For the more concrete thinker, this happenstance acts as powerful reinforcement of the idea that dice-shaking and radioactive decay obey the same type of physical laws.

Analysis and Application

In the most recent offering of this experiment, the use of the graphing calculator was also introduced. These calculators have the capacity to give a function that describes a certain set of data that has been entered into the calculator. Alternatively, certain function classes can be tested to see if a fit with the data is obtained. The same result can be obtained by using certain spreadsheet programs that are capable of giving best fits to a set of data and specifying the equation of the best fit. There is a very nice fit between the experimental results, the best fit for a logarithmic function, and the curve that would be obtained from a pure logarithmic function. Typical results for the experiment with 4-sided dice, in which a half-life of 2.7 shakes is specified, are shown in Figure 4.

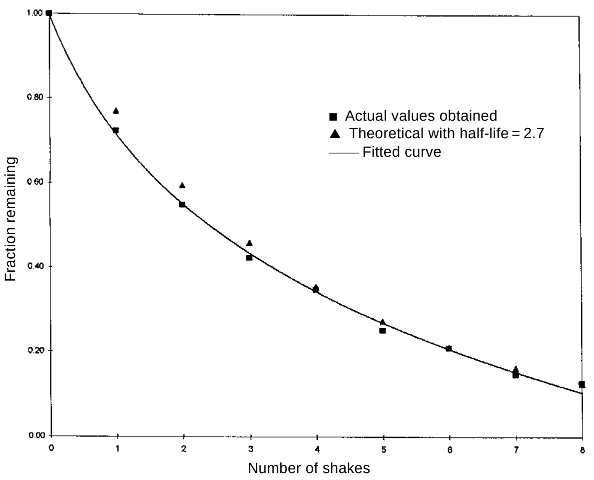

Figure 4. Comparison of actual values obtained, computer-fitted logarithmic curve, and theoretical curve for the experiment of shaking 4-sided dice. Theoretical curve obtained by specifying the rate constant obtained from a half-life of 2.7 shakes.

This experiment has been done with the following audiences: science-oriented minority high school students, general chemistry students, honors freshman (mostly nonscience), and “remedial” science students. Graphing is a central component of this experiment; it is a vital skill that my experience has told me is a serious weakness in current students that has to be addressed forcefully. The focus in all cases is the concept that radioactivity is a phenomenon that obeys the laws of probability and that nothing in the power of the experimenter can change this. The experiment is highly adaptable and the level of mathematics can be adjusted to the audience being addressed.

Two subtle differences that more perceptive students may pick up on need to be mentioned. In the dice experiment the total number of dice that are in play are being looked at. In the radioactive decay, the number of decays during a given interval *of the total amount that are potentially radioactive* are being looked at. Secondly, someone familiar with statistics recognizes that true probability is not being measured in the case of the dice-shaking experiment. The situation is one that is better described by conditional probability. The problem is that only those dice that have a potential to “decay” are left in play; the decayed dice are removed. A truer analogy would be if the decayed dice were

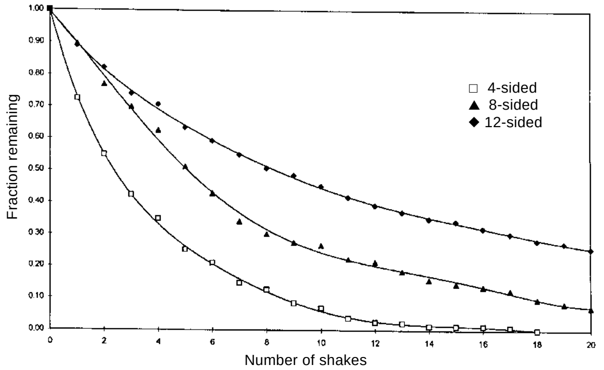

Figure 2. Decay curves for 4-, 8-, and 12-sided dice.

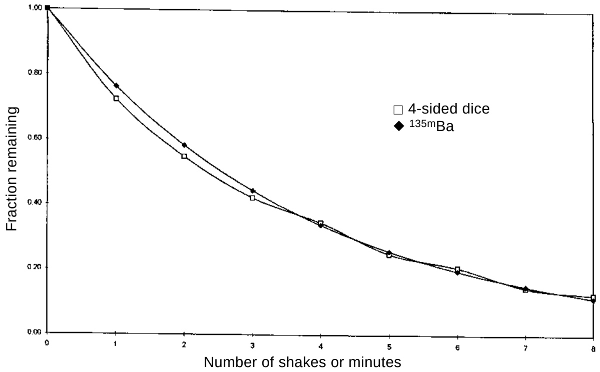

Figure 3. Decay curves for 4-sided dice and ^{137m}Ba.

labeled with a sticker and then put back into play. Only after the last of the unlabeled dice has been converted would the experiment be concluded. This approach could easily be adopted for certain audiences whereas the “simplified approach” would suffice for others.

Conclusion

The dice-shaking analogy, in addition to establishing an intuitive view of the concepts of radioactivity and half-life, provides a natural connection to the mathematical formalism and graphical treatment that describes radioactive decay. The connection to generalized first-order kinetics is just as simple. The rate of the reaction will depend upon the total concentration of reactant (dice) and the rate constant for the process (reactivity). This reactivity in dice terms could be stated as the frequency of attaining the transition state and/or the necessary activation energy. Four-sided dice would have a higher frequency than 20-sided dice. By varying the number of dice, it could easily be demonstrated that the reactivity (rate constant) is independent of the number of dice (concentration). This analogy of first-order kinetics is similar to a games approach reported by Harsch that models kinetics and mechanism using colored balls (*2*).

Probability and statistics are important ideas in chemistry, especially in quantum mechanics and entropy. The dice-shaking analogy provides a rough early intuitive appreciation of these ideas in the context of a physical process (radioactive decay) in which the connection is not only direct and easily comprehensible, but also fun.

Literature Cited

1. Hughes, E., Jr. *J. Chem. Educ.* **1991,** *68*, A41.
2. Harsch, G. *J. Chem. Educ.* **1984,** *61*, 1039.