Instructor: Dr. Daniel Short Student: ___________________________ /10

TEST 2 (of 3)

Show all of your work. Students should make use of the conversion factor method throughout and express their answers in scientific notation.

1. Fourier's law relates heat loss to thermal conductivity can be written as follows:

$$Q = \frac{k \, \Delta T}{\Delta z}$$

Where Q = heat conducted per unit area (Watt m^{-2}), k = thermal conductivity (W m^{-1} K^{-1}), Δz = length (m), ΔT = temperature difference.

(a) What is the source of the heat flux from the interior of the Earth? The global average heat flux is 0.07 W m^{-2}. What is the heat flux in milli Watts (mW m^{-2})?

(b) The average thermal conductivity of the Earth's surface is 2 - 3 W m^{-1} K^{-1} and the heat flux actually varies between 0.04 and 0.09 W m^{-2}. What is the range of average temperature gradient $\Delta T / \Delta z$ in K km^{-1}?

(c) Describe how you would calculate the total heat flow from the Earth (W) given the mean surface heat flow from each of the continents.

Average Values for Heat Flow at the Earth's Surface

Region	Mean Surface Heat Flow (mW m^{-2})
Africa	49.8
South America	52.7
North America	54.4
Australia	63.6
Europe	60.2
mean continental	56.5
North Pacific	95.4
South Pacific	77.4
North Atlantic	67.4
mean oceanic	78.2

2. The amount of radiation emitted from a black body at temperature T is given by the Stefan-Boltzman law: $E = \sigma T^4$

The dominant wavelength at which most radiation is emitted is given by Wien's law: $\lambda_{max} = 0.0029 / T$

Where E = energy (W m^{-2}), σ = Stefan's constant = 5.7×10^{-8} W m^{-2} K^{-4}, T = absolute temperature (K), and λ_{max} = wavelength of maximum emission.

(a) The dominant wavelength of light emission from the Sun is 4.83×10^{-7} m. Calculate the surface temperature of the sun in Kelvin.

(b) What is the ratio of the amount of radiation emitted by the Sun to that of the Earth (surface temperature ~15 ºC)?

(c) Place the following radiation types in order of decreasing wavelength:

- light
- X-rays
- Infra-red
- microwaves

3. A solar water heater absorbs sunlight and converts it to hot water at 35 % efficiency. The area of the panel is 6 m^2 and the sun's energy is falling at 850 W m^{-2}. How long does it take to heat 30 L of water for a shower from 20 ºC to 50 ºC?

(a) Using the equation $Q = mc\Delta T$ (where Q = heat, m = mass, c = specific heat water = 4.2 kJ kg^{-1} K^{-1}, T = temperature) to calculate the energy required to raise 30 L of water from 20 ºC to 50 ºC. (ρ_{water} = 1 kg L^{-1})

(b) Calculate the power in Watts supplied by the solar cell working at 35 % efficiency.

(c) Use the equation $E = P\,t$ (where E = energy, P = power, and t = time) to calculate the time taken to heat the water.

4. In a stream or river water flows slowly where it is wide and deep, and more rapidly where it is shallow or narrow – but the same volume of water passes each point in one second anywhere down its length. This is called the **principle of continuity of flow**.

$$Q = v\,A$$

Where Q = flow rate, v = velocity, and A = cross-sectional area.

The average flow velocity of a river is 0.5 m s^{-1} where it is 1.5 m deep and 10 m wide.

(a) What is the flow rate?

(b) Just downstream, in a narrower part of the river it is 2 m wide and 1 m deep. What is the flow rate?

(c) What is the average velocity here?

5. The **ideal gas law** is the equation of state of an ideal gas. The state of an amount of gas is determined by its pressure, volume, and temperature.

$$PV = nRT$$

Where P = pressure, V = volume, n = no. moles, R = universal gas constant = 8.314 J K^{-1} mol^{-1}, T = temperature

(a) The number of moles n of a substance is equal to the mass divided by the molecular mass ($n = m/M$). Derive the following equation, $P = R^*\rho T$ (where $R^* = R/M$ = gas constant for this gas)

$$PV = nRT$$

$$P = R^*\rho T$$

(b) If the atmosphere consisted of only nitrogen and oxygen with masses in the same ratio as the real atmosphere, what would be the apparent molecular weight ($\overline{M}$) of the mixture?

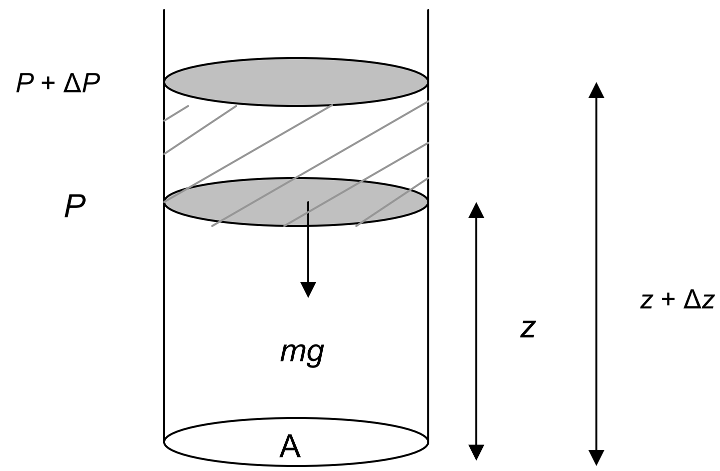
$\overline{M} = \Sigma\, r_i M_i$

Where r_i = volume mixing ratio and r_{N2} = 78.09 / 99.04, r_{O2} = 20.95 / 99.04, M_{N2} = 28.04 and M_{O2} = 32.00.

(c) Calculate R^* for dry air.

[Ans = 288 J K^{-1} kg]

BONUS:



(a) Express the mass of the element of air in terms of A the cross-sectional area of the column of atmosphere, the height of the element (Δz) and the air density (ρ).

(b) The upward pressure force on the bottom of the element from the air below = AP, downwards pressure force on top of the element from the air above = $A(P + \Delta P)$.

The net upward pressure force = $AP - A(P + \Delta P) = -A\Delta P$

This balances the force of gravity (mg).

$$mg = -A\Delta P$$

Substitute in your answer from part (a) and derive the 'hydrostatic relationship' $\Delta P / \Delta Z = -\rho g$