TEST 1 (of 3)

Show all of your work. Students should make use of the conversion factor method throughout and express their answers in scientific notation.

1. The rate of diffusion depends on the concentration gradient, that is the rate of change of concentration with distance. Fick's law states that the diffusion rate is proportional to the concentration gradient:

$$Q = \frac{k_D (C_2 - C_1)}{(z_2 - z_1)}$$

Where Q = the mass flux of material diffusing per unit area (kg m^{-2} s^{-1}), C = concentration (kg m^{-3}), and z = distance (m).

(a) Using dimensional analysis derive the SI units of the diffusion coefficient, k_D.

(b) An alternative unit for Q is molecules cm^{-2} s^{-1} when the units of concentration are molecules cm^{-3}. What are the new units for the diffusion coefficient?

(c) A gas with concentration 5×10^{12} molecules cm^{-3} diffuses over 50 m. The diffusion coefficient for the gas in air is 0.01 [units as part (b)]. Assuming typical background concentrations of the gas in air to be 2×10^{12} molecules cm^{-3} calculate the mass flux per unit area.

2. This question concerns the settling time of 2 particles of similar composition and diameters 20 μm and 10 μm.

(a) Write down an equation for an object traveling at constant velocity (*v*) relating *v* to the distance (*s*) and time (*t*).

(b) According to Stoke's law the settling velocity of particulates in the atmosphere is directly proportional to the square of the radius, *r*,

$$v_t = \frac{2\, g\, r^2\, \rho}{9\mu}$$

If the velocity is directly proportional to the square of the radius then the time required for settling is ________________ proportional to the square of the radius (bigger particles fall faster).

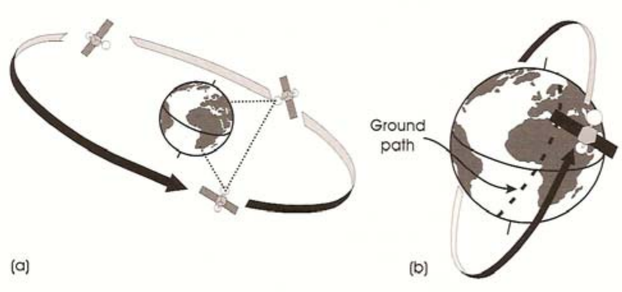
(c) The time taken for the particles to settle can be calculated as follows:

$$(t_{10} / t_{20}) = (d_{20} / d_{10})^2$$

Where *d* = diameter of the particle (2 x *r*). If the larger particle settles out in 2 days, how long will it take for the smaller particle to settle out?

3. A low earth orbit (LEO) describes a satellite which circles close to the Earth. Generally, LEOs have altitudes of around 0 - 2000 km.

(a) Name the two different types of orbits shown in the diagram below. Which orbit has the higher altitude?



(b) Consider a satellite with mass m_{sat} orbiting a planet with a mass of mass m_{planet}. If the satellite moves in circular motion, then the net centripetal force, f_c, acting upon this orbiting satellite is given by the relationship,

$$f_c = \frac{m_{sat} v^2}{R}$$

Where m_{sat} = mass of satellite, v = velocity of satellite and R = radius of orbit.

This net centripetal force must balance the gravitational force which attracts the satellite towards the central body and can be represented by,

$$f_g = \frac{G\, m_{planet}\, m_{sat}}{R^2}$$

Derive an expression for the velocity of the satellite.

[Answer: $v = \sqrt{(G\, m_{planet} / R)}$]

(c) A satellite wishes to orbit the earth at a height of 100 km above the surface of the earth. Determine the speed and acceleration of the satellite. (Given: $m_{earth} = 5.98 \times 10^{24}$ kg, $R_{earth} = 6.37 \times 10^6$ m, and $G = 6.673 \times 10^{-11}$ N m^2 kg^{-2}).

(Note: Radius of orbit = orbital height + radius of Earth)

4. A coal burning power plant produces an output of 950 MW at an efficiency of 38 %. What mass of coal (having energy content 29 MJ kg^{-1}) is required annually to operate this plant?

(a) Calculate the total power produced by the burning of the coal (Total P_{IN}) using the efficiency as follows:

$$\eta = \frac{\text{Useful } P_{OUT}}{\text{Total } P_{IN}} \times 100\%$$

(b) Determine the total energy, E, in joules (where 1 W = J s^{-1}) produced in one year.

(c) Calculate the mass of coal required.

5. Wind power currently produces just over 1% of world-wide electricity use.

(a) Explain what the following symbols mean in relation to wind power.

$$E = \frac{1}{2} \rho v^3 \pi r^2$$

(b) Location A has average annual wind speeds of 10 m s^{-1} and location B has average annual wind speeds of 8 m s^{-1}. What would a choice of location A over location B do to the amount of energy generated by a turbine?

(c) Atmospheric temperature usually decreases with altitude. Increasing the height of a wind turbine by 60 m also has a significant affect on the energy it produces. Using one of the components of the equation from part (a) explain why this occurs.

BONUS:

(a) It can be shown that the power *P* per unit length l (W m^{-1}) of wave-front in a surface wave is,

$$\frac{P}{l} = \tfrac{1}{4}\, \rho\, g^{3/2} A^2 \sqrt{(\lambda / 2\pi)}$$

Where ρ = density of water (1×10^3 kg m^{-3}), g = acceleration due to gravity (9.8 m s^{-2}) A = wave amplitude, λ = wavelength (m).

Show that this equation reduces to:

$$\frac{P}{l} = 3 A^2 \sqrt{\lambda} \qquad \text{kW m}^{-1}$$

(b) Calculate the power delivered by a 1.0 km wave front of an ocean wave with an amplitude of 1.0 m and a wavelength of 100 m.

(c) What are the pros and cons of using wave energy?