TEST 1 (of 3)

Show all of your work. Students should make use of the conversion factor method throughout and express their answers in scientific notation.

1. When large objects fall through viscous fluids skin drag dominates

$$F_d = 6\pi\, r\, \mu\, v$$

Where F_d = drag force in Newtons (1 N = kg m s^{-2}), r = radius (m), μ = viscosity (?), and v = velocity (m s^{-1}).

(a) Using dimensional analysis derive the SI units of viscosity.

$F_d = 6\pi\, r\, \mu\, v$

kg m s^{-2} = m x μ x m s^{-1}

$\mu = \dfrac{\text{kg}}{\text{m s}}$

(b) An accepted unit for viscosity is the Pascal second (Pa s), where Pascal is the unit of pressure (= force / area). Use dimensional analysis to show that the 'Pascal second' is equivalent to your answer from part (a).

$\text{Pa s} = \dfrac{\text{kg m s}^{-2}}{\text{m}^2} \times \text{s} = \dfrac{\text{kg m} \times \text{s}}{\text{m}^2\ \text{s}^2} = \dfrac{\text{kg}}{\text{m s}}$

(c) Calculate the drag force acting on a bacteria (r = 1 μm) swimming through water at a velocity of 1 m s^{-1}. Viscosity of water = 0.001 [units as above]. Give your answer in Newtons and pico Newtons (10^{-9} N).

$F_d = 6\pi\, r\, \mu\, v$

$F_d = 6 \times 3.14159\ldots \times 1 \times 10^{-6}\ \text{m} \times 0.001\ \text{kg m}^{-1}\ \text{s}^{-1} \times 1\ \text{m s}^{-1}$

$F_d = 2 \times 10^{-8}$ N = 20 pN

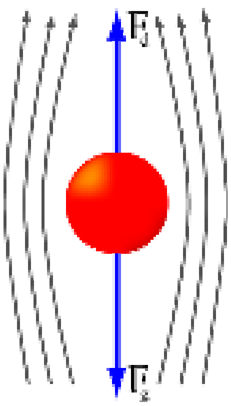
2. A lead sphere of radius 10 mm falls through viscous syrup in a glass cylinder. The terminal velocity of the sphere is achieved when the skin drag force (F_d) is equal to the downwards force of gravity (F_g).

(a) Write down an equation relating F_g to the mass of the sphere (m) and the acceleration due to gravity (g).

$F_g = mg = F_d = 6\pi\, r\, \mu\, v$

$mg = 6\pi\, r\, \mu\, v$

(b) Derive Stoke's equation for terminal velocity (v_t) by equating the two forces shown in the diagram below. Use the fact that the volume of a sphere is equal to 4/3 $\pi\, r^3$ and that mass = density x volume ($m = \rho_s\, v$).



$F_g = F_d$

$mg = 6\pi\, r\, \mu\, v$

Let $m = \rho_s\, V = \rho_s\, 4/3\, \pi\, r^3$

$\rho_s\, 4/3\, \pi\, r^3\, g = 6\pi\, r\, \mu\, v$

$$v = \frac{\rho_s\, 4/3\, \pi\, r^3\, g}{6\pi\, r\, \mu} = \frac{4\, g\, \rho_s\, r^2}{18\, \mu}$$

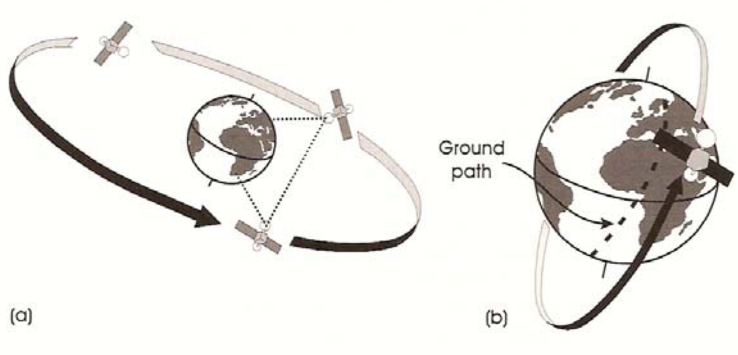
$$v_t = \frac{2g\, \rho_s\, r^2}{9\, \mu}$$

(c) Use your equation from part (b) to calculate the terminal velocity of the lead sphere. Density of lead = ρ_{lead} = 11,340 kg m^{-3}, μ = 100 Pa s, g = 10 m s^{-2}.

$$v_t = \frac{2g\, \rho_s\, r^2}{9\, \mu} = \frac{2 \text{ x } 10 \text{ m s}^{-2} \text{ x } 11{,}340 \text{ kg m}^{-3} \text{ x } (0.01 \text{ m})^2}{9 \text{ x } 100 \text{ Pa s}} = \frac{22.68 \text{ kg s}^{-2}}{900 \text{ kg m}^{-1} \text{ s}^{-1}} = 0.0252 \text{ m s}^{-1}$$

3. A low earth orbit (LEO) describes a satellite which circles close to the Earth. Generally, LEOs have altitudes of around 0 - 2000 km.

(a) Name the two different types of orbits shown in the diagram below. Which orbit has the higher altitude?



(a) = Geostationary orbit – higher altitude, (b) = polar orbit (LEO)

(b) A satellite doesn't fall straight down to the Earth because of its acceleration. Throughout a satellites orbit there is a perfect balance between the gravitational force due to the Earth, and the centripetal force necessary to maintain the orbit of the satellite.

(c) Calculate the escape velocity for a low earth orbit satellite experiencing a centripetal force:

$$F_c = mv^2/R$$

Where m = mass (kg) v = velocity (m s^{-1}), R = Earth's radius (6,400 km).

And a gravitational force $F_g = mg$.

$F_g = mg = F_c = mv^2/R$

$mg = mv^2/R$

$v = \sqrt{(gR)} = 8000$ m s^{-1}

4. A typical light-bulb consumes 50 Watts of electrical power and energy costs 8 cents per kilowatt-hour.

(a) Write down the equation relating Energy (J) to Power ($J\ s^{-1}$) and time (s).

$E = P \times t$

(b) If 1 Watt = 1 $J\ s^{-1}$ use the above equation to show that there are 3.6 MJ in 1 kWh.

1 kWh = 1 kW x 1 h
= 1000 $J\ s^{-1}$ x 3600 s
= 3600000 J
= 3.6 MJ

(c) If you use your light on for 24 hrs a day how much does it cost?

$E = P \times t$ = 50 W x 24 h = 1200 Wh = 1.2 kWh

1.2 kWh x 0.08 c/kWh = 10 cents

5. Hydropower supplies 19% of the world's energy.

(a) Explain what the following symbols mean in relation to hydropower.

$$E = P\,t = mgh$$

$$P = \frac{mgh}{t}$$

P = power, m = mass, g = gravity, h = height, t = time

(b) Derive an equation for power generated from a hydroelectric station relating power to rate of fluid flow, φ (=v/t) ($m^3\ s^{-1}$).

$P = mgh\ /\ t$

$m = \rho\ v$

$$P = \frac{\rho\, v\, g\, h}{t} = \rho\, \varphi\, g\, h$$

Density of water = 1000 kg m^{-3}

Power in W: $P = 1000\ \varphi\ g\ h$

Power in kW:

$P = \varphi\ g\ h$

[Answer: $P = \varphi\ g\ h$]

(c) Calculate the electricity generated by a micro hydroelectric station operating at a head of 10 m with a flow of 0.5 $m^3\ s^{-1}$. The efficiency of the station is 50%.

$P = 0.5\ m^3\ s^{-1} \times 10\ m\ s^{-2} \times 10\ m \times 0.5 = 25\ kW$

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BONUS:

(a) What is 'biomass', why is it considered Carbon neutral?

(b) The total available world fertile land is 8.7 billion ha. The production level of biomass is 5 t ha^{-1}.

How much biomass can be produced?

(b) Energy content of biomass is 15 GJ t^{-1}. If the entire area was given over to energy production how much energy could be generated? How would the energy produced compare to the world's current consumption (13 TW)?